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Do 10 steps. Solve exactly. Compute the error. Use 6 decimal places.

$$y' = y - y^2; \quad y(0) = 0.2, \quad h \leq 0.1$$

exact solution:

$$\frac{dy}{y - y^2} = dx$$

$$\frac{dy}{(y - \frac{1}{2})^2 - \frac{1}{4}} = -dx$$

$$\int \frac{dy}{(y - \frac{1}{2})^2 - \frac{1}{4}} = -x + C$$

$$u = y - \frac{1}{2}$$

$$du = dy$$

$$\int \frac{du}{u^2 - \frac{1}{4}} = -x + C$$

using the L.A of integrals of rational functions:

$$-\frac{2}{\sqrt{1}} \operatorname{arctanh}\left(\frac{2u}{1}\right) = -x + C \quad \left(-\frac{1}{\sqrt{1}} \operatorname{arctanh}\left(\frac{x}{1}\right) \right)$$

at $|x| < |a|$

$$2 \operatorname{arctanh}\left(\frac{x}{1}\right) = x$$

$$\text{if } y - \frac{1}{2} > 0 \Rightarrow y > \frac{1}{2}$$

$$y - \frac{1}{2} < \frac{1}{2}$$

$$y < 1$$

$$\text{if } y < \frac{1}{2}$$

$$-(y - \frac{1}{2}) \frac{dy}{y - \frac{1}{2}}$$

$$u - \frac{1}{2} > -\frac{1}{2}$$

$$u > 0$$

The bounds for the possible values of y where the exact function is correct is

$$0 < y < 1$$

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$$2u = \tanh\left(\frac{x}{2} + C\right)$$

$$\frac{2u-1}{2}$$

$$2u-1 = \tanh\left(\frac{x}{2} + C\right)$$

$$u = \frac{\tanh\left(\frac{x}{2} + C\right) + 1}{2}$$

applying initial conditions:

$$0.2 = \frac{\tanh\left(\frac{0}{2} + C\right) + 1}{2}$$

$$0.8 = \frac{\tanh(C) + 1}{2}$$

$$0.2 = \frac{\tanh\left(\frac{0}{2} + C\right)}{2}$$

$$-0.2 = \frac{\tanh(C)}{2} \Rightarrow \tanh(C) = -0.4$$

$$C = \operatorname{arctanh}(-0.4)$$

$$C = -\frac{1}{2} \ln(4)$$

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Therefore the equation of the exact function is:

$$y = \frac{\tanh\left(\frac{x}{2} - \frac{\ln(4)}{2}\right)}{2} + \frac{1}{2}$$

side note!

The integration formula used for the integration of the left hand side of

$$\int \frac{dx}{u^2 - a^2} = -\frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right)$$

if:

$$\int \frac{dx}{x^2 - a^2} = -\frac{1}{a} \operatorname{arctanh}\left(\frac{x}{a}\right) \text{ if } |x| < |a|$$

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 before we find the approximate solution,
 let us find where $1 - 0.64 e^x = 0!$

$$1 - 0.64 e^x = 0$$

$$e^x = \frac{1}{0.64} \quad \text{***}$$

$$x = -\ln(0.64)$$

$$x \approx 0.446287$$

The formula that would be used is
 as follows:

$$y_{n+1} = y_n + f(x_n, y_n) h \quad \text{h is the step size}$$

This formula is valid for equations of the
 form $\frac{dy}{dx} = P(x, y)$ with initial conditions $y(x_0) = y_0$.

For example, in our equation $\frac{dy}{dx} = y - y^2$, $y(0) = 0.2$, $h = 0.1$.

Index	x	y
0	0	0.2
1	$0+h$	$0.2 + (0.2 - 0.2^2)(0.1)$

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index (step)	x	y	batch value	error
0	0.0	0.200000	0.200000	0.000000
1	0.1	0.216000	0.216481	0.000481
2	0.2	0.232534	0.233922	0.000988
3	0.3	0.250802	0.252317	0.001515
4	0.4	0.269592	0.271645	0.002053
5	0.5	0.289243	0.291875	0.002592
6	0.6	0.309843	0.312965	0.003122
7	0.7	0.331227	0.334858	0.003631
8	0.8	0.353379	0.357486	0.004107
9	0.9	0.376229	0.380767	0.004528
10	1.0	0.399697	0.404610	0.004913